

Breaking CP and supersymmetry with orbifold moduli dynamics

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Abstract

We consider the stabilization of string moduli and resulting soft supersymmetry-breaking terms in heterotic string orbifolds. Among the results obtained are: formulae for the scalar interaction soft terms without integrating out the hidden sector gaugino condensate, which reduce to standard expressions in the usual “truncated” limit; an expression for the modular transformation of A -terms; a study of the minima of the scalar potential in the Kähler modulus direction; and a discussion of the implications for CP violation phenomenology.

This is essentially an excerpt from the author’s D. Phil. thesis, completed in September 2000, with updated references and presentational changes. Some closely related results have appeared in a recent paper of Khalil, Lebedev and Morris, namely, the exact modular invariance of A -terms up to unitary mixing, and the existence of certain complex minima for string moduli.

1 Introduction

Many phenomenologically important quantities in the low-energy effective theory of heterotic string orbifolds, including gauge and Yukawa couplings, depend on the vacuum expectation values (v.e.v.’s) of *moduli*. These are scalar fields that parameterize the metric and background fields of the compact space: their potential vanishes in perturbation theory, so it is essential

to have some mechanism for stabilizing their values. Similar remarks apply to Wilson lines that can take a continuous range of values. Once supersymmetry (SUSY) is broken, necessarily by nonperturbative effects, the moduli get, in general, a nonvanishing scalar potential, or equivalently the compactification parameters are dynamically determined [1]. If supersymmetry is broken at an energy scale far below that of the string, one should be able to study modulus stabilization using an effective field theory (see *e.g.* [2, 3]).

We will focus on the implications of the moduli dynamics for CP violation in the resulting low-energy theory of softly-broken supersymmetry. The resulting phenomenology is in general described by complex Yukawa couplings $y_{ij}^{u,d,e}$ and soft terms [4], namely the gaugino masses, bilinear Higgs couplings $B\mu H_U H_D$ and trilinear scalar couplings (“*A*-terms”) $A_{ij}^u y_{ij}^u \tilde{q}_i \tilde{u}_j^c H_U$ *etc.*. The phases of these terms are tightly constrained by experimental limits on electric dipole moments (for flavour-diagonal interactions) [5, 6, 7, and references therein] while flavour-off-diagonal interactions in the quark mass basis are constrained by experimental results for *K* and *B* mesons [8].

Whilst studies have appeared parameterizing the contribution of the moduli to CP violation in soft terms [9] by various assumptions about the magnitudes and phases of *F*-terms, our work addresses the question of how these terms, and the sources of CP violation in Yukawa couplings, are to be generated from a consistent model of SUSY-breaking and modulus stabilization. Research along similar lines has been pursued by Bailin *et al.* [10, 11, 12, 13].

The effective theory is believed to be invariant under target-space modular transformations [15, 16] for which the modulus T^i transforms as

$$T^i \rightarrow \frac{\alpha T^i - i\beta}{i\gamma T^i + \delta} \quad (1)$$

where $\alpha, \beta, \gamma, \delta$ are integers satisfying $\alpha\delta - \beta\gamma = 1$ [14]. The moduli T_i , $i = 1, 2, 3$ are defined as $iT^i = 2(B_{2i-1,2i} + i\sqrt{\det G^i})$, where i labels the three complex planes of the orbifold, B_{mn} , G_{mn} are the antisymmetric tensor field and metric respectively in the compact directions and G^i is the i 'th 2-by-2 submatrix of G_{mn} (assumed block diagonal). We simplify to the case of a single overall modulus $T^i = T$. The dilaton S [17] and matter fields [18] may also have transformation properties. This symmetry has many implications for the behaviour of the effective theory: no observable modulus-dependent quantity can change if the v.e.v. of T is replaced by its image under transformation. In addition the underlying theory is exactly CP invariant,

since CP is a discrete gauge symmetry of strings [19, 20], thus $\langle T \rangle$ and $\langle T^* \rangle$ describe formally equivalent vacua.

Then the values at which $\langle T \rangle$ is stabilized are important in finding the modulus-dependent low-energy couplings, in particular the Yukawa couplings and the scalar soft SUSY-breaking terms. Here we parameterize the unknown dilaton dynamics by assuming that the dilaton v.e.v. is constant (up to a well-defined loop correction that is required to preserve modular invariance) and varying the value of the dilaton F -term within reasonable limits. This is equivalent to assuming that the stabilization mechanism for the T -modulus has essentially no effect on the value of the dilaton, and thus that the variation of the dilaton v.e.v. and F -term can be neglected. This assumption has since been tested in [21], where essentially the same range of models for the dynamics of T was considered, but the effects of different mechanisms for dilaton stabilization were worked out in detail. The results are striking: in most cases the dilaton v.e.v. is indeed constant, despite some variation in the values of $\langle T \rangle$ obtained. Then our previous results, and those of [12, 11], remain useful, and in fact coincide with some of the phenomenologically interesting minima found in [21].

It was shown in [22] that in orbifold models with three matter generations, CP violation in Yukawa couplings cannot originate from a modulus v.e.v. on the boundary of the fundamental domain, in other words satisfying either $|\langle T \rangle| = 1$ or $\text{Im} \langle T \rangle = 1/2 + n$, n integer. This resulted from an interplay between the modular and CP invariances. Since in many cases T is stabilized at precisely these values [23], this seems to rule out many scenarios for the origin of CP violation. Recently, however, a nonzero value of the Jarlskog parameter J_{CP} , which derives from the Standard Model Yukawa couplings [24] was claimed to occur for T on the unit circle [25], in a model with more than three generations where some fields are assumed to become heavy by an unspecified mechanism. The previous result is evaded by taking the three light generations to mix with the heavy matter under a modular transformation. The calculated form of $J_{CP}(T)$ is not modular invariant, which appears suspicious from the point of view of an exactly invariant theory. In fact for $|T| = 1$, J is sent to $-J$ under the duality $T \rightarrow 1/T$, for $|T| = 1$. Minima on the unit circle are generic, so it is important to understand this result and investigate whether it is consistent with an explicit mechanism for decoupling the unwanted matter. This issue will be addressed in a future publication.

All the material presented in section 2 onwards, except the “Further reflections”, formed part of the author’s D. Phil. thesis, completed at the Uni-

versity of Sussex in September 2000. Our derivation of the modular transformation of A -terms is similar to that of Khalil *et al.* in [21], and as noted above some of our minima in the T direction coincide with theirs, although they are now able explicitly to test, and in many cases verify, our previous assumption of a constant string coupling.

2 Soft supersymmetry-breaking terms

We calculate soft supersymmetry-breaking terms explicitly for particular models motivated by perturbative heterotic string theory, and find the dependence on the dilaton and moduli. This will enable us to look in more detail at the scenario in which CP violating phases appear in the scalar trilinear A -terms and bilinear B term due to a complex v.e.v. of the T modulus. We will concentrate on the modular transformation properties of the soft terms, and the question of whether deviations from the truncated value of the condensate induce significant changes in the calculation of soft terms.

2.1 Calculation of soft terms

By coupling the visible sector, which has just the field content of the MSSM, to supergravity and a hidden sector in which supersymmetry is broken, the low-energy theory appropriate for calculating phenomenology can be found [26]. The procedure is described in more detail in, for example, [28, 29]. In principle one should find the vacuum of the full theory by minimising the scalar potential in terms of all the fields, including visible and hidden matter; in practice for strings at the Planck scale and a hidden sector with a strong interaction scale a few orders of magnitude below, the effects of the visible sector v.e.v.'s will be negligible.

To define the full theory, the superpotential and Kähler potentials for the visible and hidden sectors are simply added. The hidden sector superpotential takes the Veneziano-Yankielowicz form [30]

$$W_{\text{np}} = \frac{f_g(S, T)}{4} U - \frac{b_0}{96\pi^2} U \ln(kU) \quad (2)$$

where $b_0 \equiv -3c(G)$ is the one loop beta-function coefficient for a SYM hidden sector without matter. The gaugino condensate is described by the effective field $U \equiv \langle W^{\alpha a} W_{\alpha}^a \rangle / S_0^3$, where S_0 is the conformal compensator

in the superconformal formulation of matter-coupled supergravity [31, 32]. The hidden sector gauge kinetic function $f_g = S + \Delta(T)/(16\pi^2)$ includes a modulus-dependent threshold correction $\Delta(T)$ [33]. On “integrating out” the condensate via $\partial W/\partial U = 0$ we find the standard “truncated superpotential”

$$W^{(\text{tr})}(S, T) = \frac{b_0}{96\pi^2} U^{(\text{tr})} \quad (3)$$

where

$$U^{(\text{tr})} = e^{24\pi^2 S/b_0 - 1} (kh(T))^{-1}, \quad (4)$$

and $h(T)$ is defined by $h(T) \equiv e^{3\Delta(T)/2b_0}$. For the theory to be invariant under $\text{SL}(2, \mathbb{Z})$ modular transformations acting on T we must have the form $h(T) = \eta^{6-18\delta_{\text{GS}}/b_0}(T)H(T)^{-1}$, where $\eta(T)$ is the Dedekind eta function, δ_{GS} is a numerical coefficient associated with the cancellation of modular anomalies [17] and $H(T)$ is a modular invariant function without singularity in the fundamental domain [23].

For the visible sector we take

$$W_v = \mu_{Wij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k$$

where Φ_i are the MSSM chiral matter superfields and y_{ijk} , μ_{Wij} are respectively the coefficients of Yukawa couplings, and a possible supersymmetric bilinear coupling which is required to couple the Higgs doublets of the MSSM in order to obtain correct electroweak symmetry-breaking (the “mu-term”). For this to occur, the parameter μ , which has the dimensions of a mass, must be of the same order as the soft supersymmetry-breaking masses. This seems to require a fine-tuning of the underlying theory, since dimensionful parameters in W are naturally of the order of the fundamental scale M_P . The “mu problem” in supersymmetric phenomenology is the question of how the hierarchy between M_P and μ can be generated. Various solutions have been proposed, notably a non-minimal mixing of the two Higgs doublets in the Kähler potential [34], which has an effect analogous to the supersymmetric μ_W term, generating a Higgsino mass term and a scalar bilinear $B\mu$ term after supersymmetry-breaking. Such a term in the Kähler potential can appear naturally if the Higgs superfields are in the untwisted sector of an orbifold compactification [35], however the resulting couplings cannot easily be written down in a form which is manifestly modular invariant (see [21] for a recent discussion). For simplicity we use the alternative option, that the term $\mu H_U H_D$ is present in the superpotential at the string scale and is

tuned to be of the right order of magnitude by some unspecified mechanism. In this case we allow for modulus-dependent couplings $\mu_{ij}(T)$, $y_{ijk}(T)$, which are in general required to maintain modular invariance.

If we assume that the matter fields are in the twisted sector of an orbifold compactification, the Kähler potential is

$$K_v = \sum_i (T + T^\dagger)^{n_i} \Phi_i^\dagger \Phi_i$$

to second order in Φ_i , where n_i is the modular weight of Φ_i . The dilaton and modulus Kähler potential $\hat{K}(S, T)$ is taken to be of the form

$$\hat{K} = P(y) - 3 \ln(T + T^\dagger) \quad (5)$$

where $y = S + S^\dagger - 1/(8\pi^2) \cdot 3\delta_{\text{GS}} \ln(T + T^\dagger)$ is a modular invariant combination of fields such that $g_{\text{string}}^{-2} = \text{Re } y$. The perturbative string Kähler potential for the dilaton $K_{\text{pert}} = -\ln(y)$ has been replaced by a real function $P(y)$ which parameterizes stringy nonperturbative dynamics, which are hoped to contribute to dilaton stabilization [36], [37, 38]. We initially take the Green-Schwarz coefficients δ_{GS} to be zero to simplify the calculations, but when calculating the scalar potential for the modulus we relax this assumption. The correct form of $P(y)$ is not known, however it is possible to constrain it by looking for a stable minimum in the potential for the dilaton and requiring $P''(y) > 0$ to obtain the right sign kinetic term. The complete hidden sector Kähler potential is then taken to be

$$K_h(U, S, T) = \hat{K} - 3 \ln \left(1 - A e^{\hat{K}/3} (UU^\dagger)^{1/3} \right) \quad (6)$$

where A is a constant. It was shown in [31] that this expression for the Kähler potential of U has the correct dependence on S and T , as well as being modular invariant. However, it can only be determined up to a constant factor, and may receive higher-order corrections. The constant A cannot at present be computed, due to our incomplete knowledge of supersymmetric gauge dynamics, but is expected to be of order unity.

The trilinear A -terms and the scalar bilinear B term corresponding to y_{ijk} and μ_{ij} respectively can be extracted from the standard formula for the scalar potential

$$V = e^K \left((W_I^* + K_I W^*) (K^{-1})_J^I (W^J + K^J W) - 3|W|^2 \right) \quad (7)$$

where the indices run over U, S, T and ϕ_i (ϕ_i being the scalar component of Φ_i) and we assume a single hidden sector gauge group. We find a general formula for soft SUSY-breaking scalar interactions:

$$\begin{aligned}\delta\mathcal{L}_{(scalar)} &= -\frac{b_0 z^{*3} e^{\hat{K}/2}}{96\pi^2(1-A|z|^2)^2} \left\{ W_v \cdot \left[\frac{P'(y)}{P''(y)} \left(P'(y) - \frac{\omega'(S^*)}{\omega(S^*)} \right) + (T+T^*) \frac{h'(T^*)}{h(T^*)} \right] \right. \\ &+ \sum_i \phi_i \frac{\partial W_v}{\partial \phi_i} \cdot \left[n_i \left(1 + \frac{1}{3}(T+T^*) \frac{h'(T^*)}{h(T^*)} \right) - \frac{\ln(e^{-\hat{K}/2} z^{*3} \omega(S^*) h(T^*))}{1-A|z|^2} \right] \\ &+ \left. \frac{\partial W_v}{\partial T} \cdot -(T+T^*) \left(1 + \frac{1}{3}(T+T^*) \frac{h'(T^*)}{h(T^*)} \right) \right\} + h.c.\end{aligned}$$

where we include terms up to third order in the visible sector fields and to all orders in the dilaton, modulus and hidden sector condensate z defined by $z^3 = e^{\hat{K}/2} U$, taking $\delta_{GS} = 0$. The function $\omega(S) \equiv e^{-24\pi^2 S/b_0}$ is defined analogously to $h(T)$.

If the visible sector superpotential contains a coupling $y_{123}(T)\Phi_1\Phi_2\Phi_3$ then the Lagrangian will contain the trilinear interaction

$$\begin{aligned}A_{123}y_{123}\phi_1\phi_2\phi_3 &= -y_{123}\phi_1\phi_2\phi_3 \frac{b_0}{96\pi^2} \frac{e^{\hat{K}/2} z^{*3}}{(1-A|z|^2)^2} \left[\frac{P'(y)}{P''(y)} \left(P'(y) - \frac{\omega'(S^*)}{\omega(S^*)} \right) \right. \\ &+ \left(3 + (T+T^*) \frac{h'(T^*)}{h(T^*)} \right) \left(1 + \sum_{i=1}^3 \frac{n_i}{3} - \frac{(T+T^*)}{3} \frac{y'_{123}(T)}{y_{123}(T)} \right) \\ &+ \left. 3 \left(-\frac{\ln(e^{-\hat{K}/2} z^{*3} \omega(S^*) h(T^*))}{1-A|z|^2} - 1 \right) \right].\end{aligned}\quad (8)$$

The auxiliary fields F^S and F^T as defined by

$$F^i = e^{K/2} (K^{-1})^i_j (W^j + K^j W). \quad (9)$$

are

$$\begin{aligned}F^S &= \frac{b_0}{96\pi^2} |z|^3 (1-A|z|^2)^{-1/2} \frac{1}{P''(y)} \left(P'(y) - \frac{\omega'(S)}{\omega(S)} \right), \\ F^T &= -\frac{b_0}{96\pi^2} |z|^3 (1-A|z|^2)^{-1/2} \frac{(T+T^*)}{3} \left(3 + (T+T^*) \frac{h'(T)}{h(T)} \right).\end{aligned}$$

Superscripts denote holomorphic indices, subscripts antiholomorphic ones, so $F_T = (F^T)^*$. The A-term can then be written as

$$A_{123} = -\left(\frac{z^*}{|z|}\right)^3 e^{K/2} \left[\hat{K}^S F_S + \hat{K}^T F_T \left(1 + \sum_{i=1}^3 \frac{n_i}{3} - \frac{T+T^*}{3} \frac{y'_{123}(T)}{y_{123}(T)} \right) \right]$$

$$+3 \frac{|b_0|}{96\pi^2} \frac{|z|^3}{(1-|z|^2)^{1/2}} \left(-\frac{\ln(e^{-\hat{K}/2} z^{*3} \omega(S^*) h(T^*))}{1-A|z|^2} - 1 \right) \Bigg].$$

Similarly, for a coupling $\mu_{12}(T) \Phi_1 \Phi_2$ in W_v a corresponding soft supersymmetry-breaking coupling $B\mu_{12}\phi_1\phi_2$ is generated, with

$$\begin{aligned} B = & -\frac{b_0}{96\pi^2} \frac{e^{\hat{K}/2} z^{*3}}{(1-A|z|^2)^2} \left[-1 + \frac{P'(y)}{P''(y)} \left(P'(y) - \frac{\omega'(S^*)}{\omega(S^*)} \right) \right. \\ & + \left(3 + (T+T^*) \frac{h'(T^*)}{h(T^*)} \right) \left(1 + \sum_{i=1}^2 \frac{n_i}{3} - \frac{(T+T^*)}{3} \frac{\mu'_{12}(T)}{\mu_{12}(T)} \right) \\ & \left. + 2 \left(-\frac{\ln(e^{-\hat{K}/2} z^{*3} \omega(S^*) h(T^*))}{1-A|z|^2} - 1 \right) \right]. \end{aligned} \quad (10)$$

In these expressions the truncated approximation for z , equivalent to imposing $U = U^{(\text{tr})}$ (eq. (4)), is implemented by neglecting $A|z|^2$ next to 1 and setting the logarithm in the second line equal to (-1) ; we can also use the formula for the gravitino mass $m_{3/2} \equiv e^{K/2}|W| = |b_0 z_{\text{tr}}^3|/(96\pi^2)$ to simplify the prefactors and make contact with established results. Deviations from the truncated approximation are treated in detail in [39] (see also [40]).

2.2 Phenomenological discussion

The complex phases and flavour structure of the soft breaking terms are mainly determined by the dependence on the T modulus: the F_S terms are universal and, for a single condensate, real. Apart from the phase of z^{*3} (which is eliminated by the redefinition of fields in going to the softly broken globally supersymmetric theory, for which see below) a complex phase can only enter through the auxiliary field F^T and the modulus-dependent couplings $y(T)$, $\mu(T)$. When T is stabilized at a minimum of the scalar potential (section 3), F^T may be zero, real or have a complex phase which is of order 0.1–1. If the term involving the derivative of $y_{123}(T)$ were absent, the A -terms would have a common phase, that of F^T , and their magnitudes would be determined by the modular weights n_i . Using an explicit formula for Yukawa couplings, Khalil *et al.* [21] recently showed that these logarithmic derivatives were real to a good approximation in some cases of interest, thus a common phase for A -terms may be a good approximation.

In the special case of all n_i equal we would recover the “minimal supergravity” ansatz for the soft terms, in which the trilinear couplings are

proportional to y_{ijk} , *i.e.* $A_{ijk} = A$ for all i, j, k . However in general the A -terms will be non-universal, due to the different values of n_i and the terms involving y'_{ijk}/y_{ijk} (which are also essential for modular invariance): their magnitudes and phases will be different and there will be *off-diagonal* (and complex) A -terms in the super-KM basis. Similarly the $B\mu$ bilinear coupling may have a phase different from that of μ (only the phase difference between $B\mu$ and μ is physically observable) which will feed through into a complex mass matrix for charginos and neutralinos at low energies. This is a phenomenologically interesting scenario, which may result in predictions for CP-violating observables which differ significantly from the SM. However it is severely restricted by the non-universality of scalar masses which would result if the matter generations of the MSSM had different modular weights: any departure from degeneracy of scalar masses is likely to result in contributions to flavour changing neutral current processes in excess of the experimental limits (see *e.g.* [8]).

In the light of this discussion, deviations from the truncated approximation in the formula (8) do not have an important direct effect. New complex phases are not introduced and the corrections arising from $z \neq z_{\text{tr}}$ are universal, *i.e.* flavour-independent. However the overall magnitude of the soft terms may be slightly changed, a “second-order” effect. So from now on we will use the standard formulas resulting from the truncated approximation:

$$A_{123} = - \left(\frac{z^*}{|z|} \right)^3 e^{\hat{K}/2} \left[\hat{K}^S F_S + \hat{K}^T F_T \left(1 + \sum_{i=1}^3 \frac{n_i}{3} - \frac{T + T^*}{3} \frac{y'_{123}(T)}{y_{123}(T)} \right) \right] \quad (11)$$

and

$$B = \left(\frac{z^*}{|z|} \right)^3 e^{\hat{K}/2} \left[-m_{3/2} - \hat{K}^S F_S - \hat{K}^T F_T \left(1 + \sum_{i=1}^2 \frac{n_i}{3} - \frac{T + T^*}{3} \frac{\mu'_{12}(T)}{\mu_{12}(T)} \right) \right] \quad (12)$$

where the auxiliary fields are now

$$\begin{aligned} F^S &= -m_{3/2} \frac{1}{P''} \left(P' - \frac{\omega'(S)}{\omega(S)} \right), \\ F^T &= m_{3/2} \frac{(T + T^*)}{3} \left(3 + (T + T^*) \frac{h'(T)}{h(T)} \right). \end{aligned} \quad (13)$$

The corresponding formulae in the case $\delta_{\text{GS}} \neq 0$, which will be needed when discussing the effect of changing δ_{GS} on the minimisation of T and on the

soft terms, have been derived by Bailin *et al.* [10]. As is well known, not all complex couplings in the Lagrangian result in CP violation. We must consider whether the phases are physical and how many can be eliminated by redefinition of fields. In particular, the behaviour of the A -terms under $\text{SL}(2, \mathbb{Z})$ modular transformations must be found, since physical quantities should be modular invariant and (in general) the individual couplings will not be. Little can be deduced from the phase of a single A -term without considering the whole set of couplings. The results (8, 11) have nontrivial modular transformations, particularly when the observable matter fields are mixed by modular transformations. Since we suppose only a single μ -term in the superpotential, modular transformations cannot mix the bilinear $B\mu$ coupling with any other and it is relatively easy to verify that this term is modular invariant. So we will focus on the trilinear A -terms.

2.3 Modular transformations of soft terms

Recall that the superpotential is a modular form of weight -3 , so under modular transformations

$$U \mapsto \frac{\zeta_W U}{(i\gamma T + \delta)^3}, \quad W_v \mapsto \frac{\zeta_W W_v}{(i\gamma T + \delta)^3}.$$

where ζ_W is a phase depending only on the parameters $\alpha, \beta, \gamma, \delta$ of the transformation. The observable fields transform into one another under $\text{SL}(2, \mathbb{Z})$ as

$$\Phi_i \mapsto C_{im}(i\gamma T + \delta)^{n_i} \Phi_m$$

where C_{im} is a unitary matrix and $n_i = n_m$ for all i, m such that $C_{im} \neq 0$, thus the Yukawa couplings are constrained to transform as

$$y_{ijk}(T) \mapsto (i\gamma T + \delta)^{(-\sum n_i - 3)} \tilde{y}_{ijk}(T)$$

where

$$\tilde{y}_{ijk} C_{im} C_{jn} C_{kp} = \zeta_W y_{mnp}$$

and $\sum n_i$ denotes the sum over the three indices, *i.e.* $(n_i + n_j + n_k)$. This implies that

$$\tilde{y}_{ijk}(T) = \zeta_W y_{mnp}(T) C_{mi}^\dagger C_{nj}^\dagger C_{pk}^\dagger.$$

We rewrite the scalar trilinear interactions as

$$A_{ijk} y_{ijk} \phi_i \phi_j \phi_k = -e^{\tilde{K}/2} \frac{z^{*3}}{|z|^3} \phi_i \phi_j \phi_k$$

$$\cdot \left[\hat{K}^S F_S y_{ijk} + \hat{K}^T F_T \left(\left(1 + \frac{\sum n_i}{3} \right) y_{ijk}(T) - \frac{(T + T^*)}{3} y'_{ijk}(T) \right) \right].$$

The quantity $\hat{K}^T F_T = b_0/(96\pi^2)|z_{\text{tr}}|^3(3 + (T + T^*)h'(T)/h(T))$ transforms as

$$\hat{K}^T F_T \mapsto \frac{(-i\gamma T^* + \delta)^2}{|i\gamma T + \delta|^2} \hat{K}^T F_T$$

where we have used the fact that $h(T)$ must have modular weight 3. Also, under modular transformations the expression

$$(1 + \sum n_i/3)y_{ijk} - \frac{1}{3}(T + T^*)y'_{ijk}(T)$$

goes to

$$\zeta_W C_{mi}^\dagger C_{nj}^\dagger C_{pk}^\dagger (i\gamma T + \delta)^{-3 - \sum n_i} \frac{(i\gamma T + \delta)^2}{|i\gamma T + \delta|^2} \left(\left(1 + \frac{\sum n_m}{3} \right) y_{mnp}(T) - \frac{(T + T^*)}{3} y'_{mnp}(T) \right)$$

since the matrices C_{ij} and the phase ζ_W do not depend on T . Recalling that z is defined as $(e^{\tilde{K}/2} U)^{1/3}$, we have

$$\frac{z^{*3}}{|z|^3} \mapsto \zeta_W^* \left(\frac{-i\gamma T + \delta}{|i\gamma T + \delta|} \right)^{-3} \frac{z^{*3}}{|z|^3}$$

under modular transformations. It is now easy to confirm that the expression $A_{ijk} y_{ijk} \phi_i \phi_j \phi_k$ is modular invariant, since $C_{mi}^\dagger C_{ij} = \delta_{mj}$ and all phases and factors of $(i\gamma T + \delta)$ cancel.

This also implies that $A_{ijk} y_{ijk}$ (no sum!) transforms “inversely” to $\phi_i \phi_j \phi_k$:

$$(Ay)_{ijk} \mapsto (i\gamma T + \delta)^{-\sum n_i} (Ay)_{mnp} C_{mi}^\dagger C_{nj}^\dagger C_{pk}^\dagger.$$

To clarify notation here we write $(Ay)_{ijk} \equiv A_{ijk} y_{ijk}$, where no sum is implied on the RHS. Then dividing by y_{ijk} and its modular transform we find

$$A_{ijk} \mapsto (i\gamma T + \delta)^3 \zeta_W^{-1} \frac{\sum_{mnp} (Ay)_{mnp} C_{mi}^\dagger C_{nj}^\dagger C_{pk}^\dagger}{\sum_{m'n'p'} y_{m'n'p'} C_{m'i}^\dagger C_{n'j}^\dagger C_{p'k}^\dagger}. \quad (14)$$

If we ignore the unitary mixings C_{ij} by assuming $C_{ij} = e^{i\theta_i} \delta_{ij}$ then all the phases θ_i cancel by inspection and the A -terms transform with a universal factor $(i\gamma T + \delta)^3 \zeta_W^{-1}$.

2.4 Rescaling to a global SUSY with soft breaking terms

The formulas presented so far have been in terms of fields normalised as in the effective supergravity theory. It is usual to rescale the visible sector component fields and rotate away the phase of the hidden sector superpotential, so that the low-energy theory is just the MSSM with *canonical* kinetic terms for the chiral matter, and soft breaking terms expressed in terms of the gravitino mass $m_{3/2} = |b_0/(96\pi^2)z^3|$ [26]. The scalar potential is then written in terms of normalised fields $\hat{\phi}_i = K_i^{1/2}\phi_i$, where $K_i(T) = (T + T^*)^{n_i}$. The Yukawa couplings are rescaled as

$$\hat{y}_{ijk} = \frac{W^*}{|W|} e^{\hat{K}/2} (K_i K_j K_k)^{-1/2} y_{ijk} \quad (15)$$

so that

$$\hat{y}_{ijk} \hat{\phi}_i \hat{\phi}_j \hat{\phi}_k = \frac{W^*}{|W|} e^{\hat{K}/2} y_{ijk} \phi_i \phi_j \phi_k$$

which is modular invariant. We have $W^*/|W| = (z^*/|z|)^3$ *in vacuo*, and since

$$A_{ijk}^{\text{SUGRA}} y_{ijk} \phi_i \phi_j \phi_k = A_{ijk}^{\text{SUSY}} \hat{y}_{ijk} \hat{\phi}_i \hat{\phi}_j \hat{\phi}_k$$

we deduce that

$$\begin{aligned} A_{ijk}^{\text{SUSY}} &= \frac{|z|^3}{z^*{}^3} e^{-K/2} A_{ijk}^{\text{SUGRA}} \\ &= -\hat{K}^S F_S - \hat{K}^T F_T \left(1 + \frac{n_i + n_j + n_k}{3} - \frac{(T + T^*)}{3} \frac{\partial \ln y_{ijk}}{\partial T} \right) \\ &= m_{3/2} \left[\frac{P'}{P''} \left(P' - \frac{\omega'(S^*)}{\omega(S^*)} \right) + \left(3 + (T + T^*) \frac{h'(T^*)}{h(T^*)} \right) \left(1 + \frac{\sum n_i}{3} - \frac{(T + T^*)}{3} \frac{y'_{ijk}(T)}{y_{ijk}(T)} \right) \right] \end{aligned} \quad (16)$$

has modular weight zero. In particular, the transformation of the factors $W^*/|W|$ and $e^{\hat{K}/2}$ cancels the previously-noted factor of $(i\gamma T + \delta)^3 \zeta_W^{-1}$, so the expression is exactly invariant if unitary mixings are neglected.

However, the A^{SUSY} still in general transform with the complicated mixing in terms of the y_{ijk} and C_{im} as indicated in (14). These involved transformation properties are a major obstacle to finding the implications of complex phases in the A -terms for physical CP-violating quantities. For a particular

v.e.v. of T there will be certain predictions for the complex phases of the A -terms, but in general these predictions will not be invariant under the modular transformation acting on T and on the matter fields. We expect that physical quantities should depend on combinations of the couplings in \mathcal{L} that are modular invariant, however in the absence of a realistic model, and in the general case where “modular eigenstates” cannot be defined, it is not clear how to construct the relevant quantities.

3 Stabilizing string moduli

3.1 Introduction

The breaking of local supersymmetry leads to a non-vanishing scalar potential for the “flat directions” of string theory, the dilaton and compactification moduli. This suggests the possibility that these quantities are dynamically determined after supersymmetry is broken. When supersymmetry-breaking is mediated by gravity the scalar potential $V(S, T)$ is of order $|F|^2$, where F denotes a supersymmetry-breaking auxiliary field, while the values of the dilaton and moduli fields vary over the Planck scale (when the units of S and T are restored) so the flat directions acquire masses of order $m_{3/2} \sim \text{TeV}$. While this scenario is not without drawbacks for cosmology, and in most models predicts a large negative cosmological constant at the minimum of V , it appears more promising for phenomenology, since the dynamics of moduli can be studied through an effective field theory and it is possible in principle to make predictions based on specific supersymmetry-breaking mechanisms.

In the models of supersymmetry-breaking via gaugino condensation that we have considered, the dependence of the scalar potential on the dilaton and moduli is determined by the gauge kinetic function of the hidden sector gauge group and the Kähler potential for S and T . We consider the simplest hidden sector consisting of a single gauge group factor without matter, however we allow the Green-Schwarz coefficient to be non-zero. For convenience we quote the scalar potential from [41, 42, 39] (*without* assuming the truncated approximation):

$$V = \left(\frac{b_0}{96\pi^2} \right)^2 \frac{|z|^4}{(1 - A|z|^2)^2} \left\{ \frac{3}{A} \left| 1 + \ln \left(\omega(S) h_1(T) e^{-\hat{K}/2} z^3 \right) \right|^2 + C'_2(S, T) |z|^2 \right\} \quad (17)$$

where

$$z = (e^{\hat{K}/2} U)^{1/3} = e^{P(y)/6} (T + T^*)^{-1/2} U^{1/3}$$

as before, and

$$\begin{aligned} C'_2(S, T) = & \left(\frac{1}{P''(y)} \left| P'(y) - \frac{\omega'(S)}{\omega(S)} \right|^2 + \right. \\ & \left. + \frac{1}{3(1 + \frac{\delta_{\text{GS}}}{8\pi^2} P'(y))} \left| 3 \left(1 - \frac{3\delta_{\text{GS}}}{b} \right) + (T + T^*) \frac{h'_1(T)}{h_1(T)} \right|^2 - 3 \right). \end{aligned} \quad (18)$$

Recall that $\omega(S) = e^{-24\pi^2 S/b_0}$, and we take the ansatz

$$h_1(T) = \eta^{6-18\delta_{\text{GS}}/b_0}(T) (H(T))^{-1}$$

where $H(T)$ is in general a modular invariant function without singularities in the fundamental domain. This form for h_1 is supposed to originate from the threshold correction

$$\Delta_a(T) = -(b_0 - 3\delta_{\text{GS}}) \ln (|\eta^4(T)|(T + T^*)) + (b_0/3) \ln |H(T)|^2,$$

with H a holomorphic function of T .

Note that modular invariant, so-called “universal” threshold corrections in heterotic string theory have been calculated [43, 44] which take the form

$$\Delta_a = -(b_0 - 3\delta_{\text{GS}}) \ln (|\eta^4(T)|(T + T^*)) - k_a Y$$

where $Y(T)$ is modular invariant but *not* the real part of a holomorphic function. As with the threshold corrections involving $\eta(T)$, the non-holomorphic part of the threshold correction appears in the T -dependence of the Kähler potential of the effective field theory, so the universal threshold corrections imply a correction to both the superpotential and Kähler potential for the dilaton and modulus. This results in the modular transformation properties becoming considerably more complicated; so these corrections cannot be included in the above prescription for $h_1(T)$. Note that the universal corrections $-k_a Y$ can be absorbed by a redefinition of the string coupling $g_{\text{string}}^{-2} = \text{Re } y$, which formally justifies neglecting them if they are small. The effect of the universal threshold corrections on stabilizing T has been found using the truncated approximation for the gaugino condensate [13] resulting

in a T -dependent potential similar in form to $V^{(tr)} \propto C'_2 |z_{tr}|^6$. We might then anticipate that the full condensate-dependent potential would have a similar form to (17) with a different function of S and T replacing C'_2 . However the mathematical complexity of the threshold corrections prevented us from investigating further.

The invariance of the theory under target-space T-duality $T \mapsto 1/T$ will ensure that the modulus is stabilised at a value of order unity (assuming that the extra dimensions are indeed dynamically compactified, *i.e.* that V becomes large and positive for $\text{Re}(T)$ very large and very small). However, in heterotic string theory no such duality applies to the dilaton; without careful model-building either the potential will have no minimum in the S -direction, or the minimum will lead to an unrealistic value of the gauge coupling. This is the well-known dilaton runaway problem, first noticed in the case of a single condensing gauge group with the Kähler potential for the dilaton taking its string tree-level form. There have been various proposals for solving it: the simplest, for our purposes, is “Kähler stabilisation” [38] where the dilaton Kähler potential is supposed to receive large corrections from nonperturbative string effects [36]. We have assumed that this is the case in deriving the formulas (17) for the scalar potential. However, since we are mainly interested in finding the v.e.v. of the T modulus, the details of the dilaton stabilization will be neglected as far as is reasonable.

3.2 Calculation procedure

We will assume that the v.e.v. of the dilaton is fixed by nonperturbative effects irrespective of the value of T , such that the unified gauge coupling takes a phenomenologically reasonable value. The (modular invariant) quantity $y = S + S^* - 1/(8\pi^2) \cdot 3\delta_{GS} \ln(T + T^*)$ will be set equal to 4, which will fix the value of S at any given value of T . The scalar potential also depends on the function $P(y)$ and its derivatives at the point $y = 4$. We will treat $P'(y)$ and $P''(y)^{-1} |P'(y) - \omega'(S)/\omega(S)|^2 \equiv V_{\text{dil}}$ as *independent* parameters and take $P(y) = -\ln 4$, $P'(y) = -1/4$ (the same values as for the perturbative Kähler potential $K_p = -\ln y$) since the effects of changing these two quantities on the potential for T are small. Specifically, changing $P(y)$ would change the overall scale of the condensate z but not the shape of the potential, while changing $P'(y)$ would have a small effect on the prefactor of the second term in (18). However, V_{dil} , the first term of C'_2 , corresponding physically to the amount of supersymmetry-breaking originating from the dilaton dynamics,

will be treated as a free (positive semidefinite!) constant parameter. It is proportional to $|F^S|^2$ and may have an important effect on the shape of the potential and the cosmological constant.

To determine the shape of the potential as a function of T , a number of further parameters have to be specified. Apart from $P(y)$ and its derivatives, we require the values of b_0 , A , δ_{GS} and the functional form of $H(T)$ specified by the integers m and n and the polynomial $p(J)$ in the expression

$$H(T) = (J - 1)^{m/2} J^{n/3} p(J).$$

which is the most general invariant form with no singularities at finite T [23]. Of these, b_0 and δ_{GS} can be calculated in specific orbifold models for a given hidden sector gauge group; however, we will take the phenomenological liberty of varying b_0 and δ_{GS} over typical ranges of values, for the purpose of illustration. The constant A is not known and will be set to unity. The form of H , which parameterizes unknown modular invariant threshold corrections, is essential to finding the minimum of the potential. In the absence of definite results from string theory, we look at a range of values for m, n and the simplest possibilities for $p(J)$, in an approach similar to [23, 10, 11], and look at the possible implications for CP violation when T is stabilized at a minimum of the scalar potential. In addition, we are able to look at the effects of deviations from the truncated approximation, which may be important if they change the position of the minimum in the complex T plane. This was not possible in the formalism of [10, 11]. We calculate the scalar potential in the truncated approximation

$$V^{(\text{tr})} = \left(\frac{b_0}{96\pi^2} \right)^2 C_2' |z_{\text{tr}}|^6 \quad (19)$$

where z_{tr} and C_2' are functions of S and T , and then find the corrections by minimising the full scalar potential in the z -direction for each value of T , with S being fixed as described above. Note that the only dependence on z_{tr} is in the overall scale of the potential.

3.3 Results: no dependence on $J(T)$

The simplest case occurs when the T -dependent holomorphic threshold corrections do not involve the absolute modular invariant $J(T)$, in other words $H = \text{constant}$. This is the form that results from a direct perturbative string

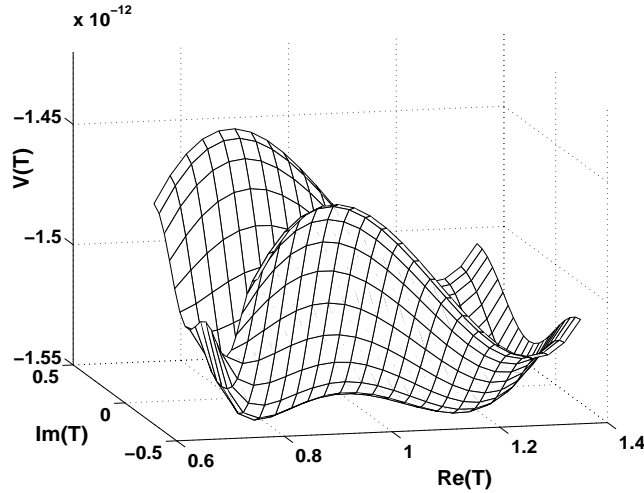


Figure 1: Scalar potential for $m = n = 0$, $p(J) = 1$, $\beta \equiv b_0/(16\pi^2) = -0.3$, $V_{\text{dil}} = 0 = \delta_{\text{GS}}$.

calculation [33], which may however miss universal, modular invariant contributions. The T -dependence of the scalar potential is well-known in this case [2, 3], however we are able to look in more detail at the effect of changing various parameters.

For $V_{\text{dil}} = 0$, corresponding to no supersymmetry-breaking in the dilaton sector, the minima of the scalar potential lie along the real axis. At $\delta_{\text{GS}} = 0$ the minima are at approximately $T = 0.8$, $T = 1.22$ (fig. 1), as δ_{GS} increases the minima approach $T = 1$ and merge (fig. 2) while at negative δ_{GS} the minima become more widely separated: at $\delta_{\text{GS}} = -10$ minima occur at $T \simeq 0.4$, 2.5 (fig. 3). Note that the minima are related by the modular symmetry $T \mapsto 1/T$; there are also minima at the modular images under $T \mapsto T + i$, which are not shown. We have also checked numerically that the scalar potential is an invariant function under the full $\text{SL}(2, \mathbb{Z})$ modular group. The corrections to the truncated approximation (at $\delta_{\text{GS}} = 0$) are shown in fig. 4. As discussed in [39] they have a T -dependence different from that of the truncated scalar potential, so in principle the corrections could alter the position of the minima. In this case the effect is not significant for phenomenology since the minima will remain on the real axis and at T -values of order unity.

When $V_{\text{dil}} = 3$, the cosmological constant is fine-tuned to zero at the minimum of the scalar potential (fig. 5). In this case, there are two degenerate

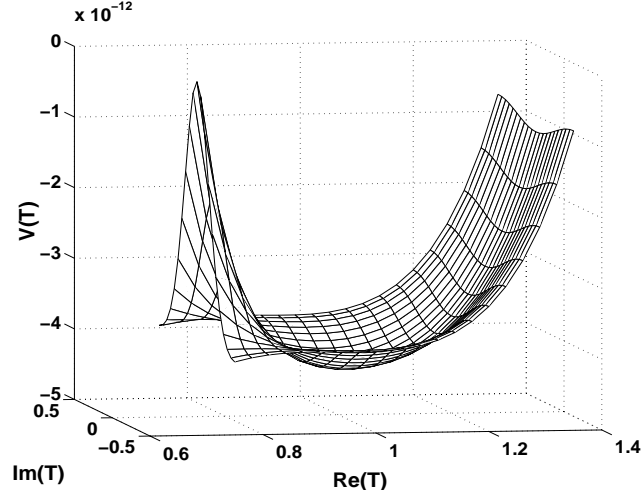


Figure 2: Scalar potential for $m = n = 0$, $p(J) = 1$, $\beta = -0.3$, $V_{\text{dil}} = 0$, $\delta_{\text{GS}} = 15$.

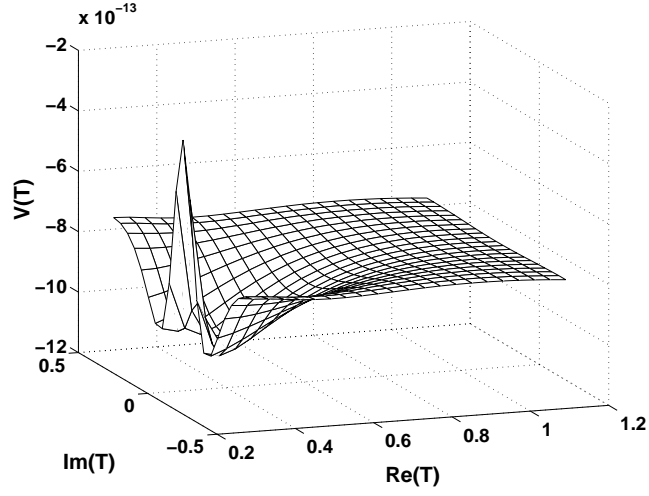


Figure 3: Scalar potential for $m = n = 0$, $p(J) = 1$, $\beta = -0.3$, $V_{\text{dil}} = 0$, $\delta_{\text{GS}} = -10$.

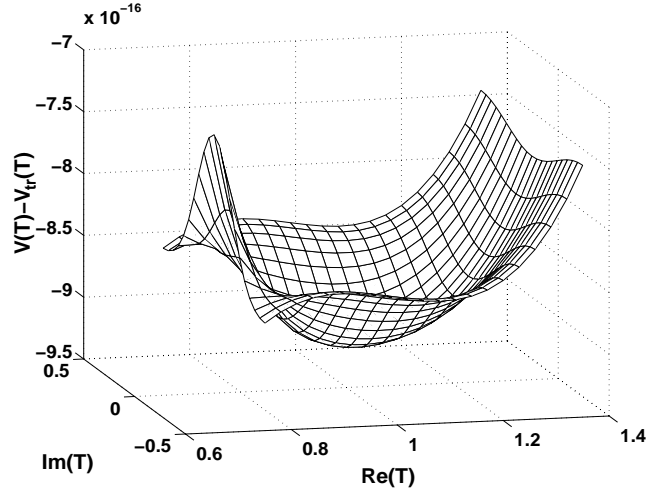


Figure 4: Corrections to the truncated potential for $m = n = 0$, $p(J) = 1$, $\beta = -0.3$, $V_{\text{dil}} = 0$, $\delta_{\text{GS}} = 0$.

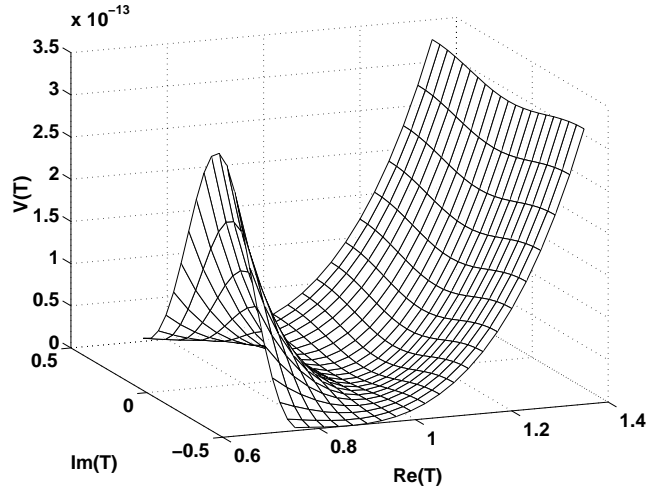


Figure 5: Scalar potential for $m = n = 0$, $p(J) = 1$, $\beta = -0.3$, $V_{\text{dil}} = 3$, $\delta_{\text{GS}} = 0$.

minima inside \mathcal{F} at $T = 1, \rho$ (where $\rho = e^{i\pi/6}$) with $C_2 = 0$ at these points, and the positions of the minima are unaffected by changing δ_{GS} . Since the corrections to the truncated approximation vanish at $C_2 = 0$, in this special case the minima are also unaffected by the corrections.

We also take $V_{\text{dil}} = 6$, which corresponds to a large, positive cosmological constant. The minima are now at $T = \rho$ and its images under modular transformation. As in the case where $V_{\text{dil}} = 3$, the position of the minimum is not changed by taking $\delta_{\text{GS}} \neq 0$, and the effect of the corrections to the truncated approximation is small. We can conclude that when the Dedekind eta function is the only modular form arising in the threshold corrections, either T is real at the minimum, in which case CP-violating phases vanish, or $T = \rho$, in which case F^T vanishes and the moduli do not contribute to supersymmetry-breaking. While this scenario is satisfactory as a solution to the supersymmetric CP problem (and, when $F^T = 0$, the supersymmetric flavour problem also) it does not make any characteristic predictions for CP-violating quantities different from those of the standard model. If there are no other sources of CP violation which can generate a CKM phase then the scenario is, of course, ruled out. Neither does it throw light on the problems of CP violation in the SM, namely the origin of the cosmological baryon asymmetry and the high value of ϵ'_K/ϵ_K .

3.4 Results for threshold corrections including $J(T)$

3.4.1 $m = 1, n = 0, p(J) = 1$

First we take the case when the function $H(T)$ is just proportional to $(J(T) - 1)^{1/2}$. This is in some sense natural, since $J - 1 \propto (T - 1)^2$ near $T = 1$, so the square root remains well-defined near the zero of H . However, as discussed in [23], the effective action for the condensate may become ill-defined at some finite radius around $T = 1$, since some string states are supposed to become light here; we also expect the truncated approximation to fail badly near $T = 1$ since the quantity $x = AC'_2|z_{\text{tr}}|^2$ which measures the size of deviations from the truncated approximation becomes large [39]. We first present the form of the scalar potential for $V_{\text{dil}} = 0 = \delta_{\text{GS}}$ (fig. 6). Note the “dimple” near $T = 1$ which is related to the failure of the truncated approximation. We can look at this area of the complex T plane in the limit where $T \rightarrow 1$ and find that while the truncated condensate value $|z_{\text{tr}}|$ vanishes as the cube root of $(T - 1)$, C'_2 diverges as $(T - 1)^{-2}$, so the truncated

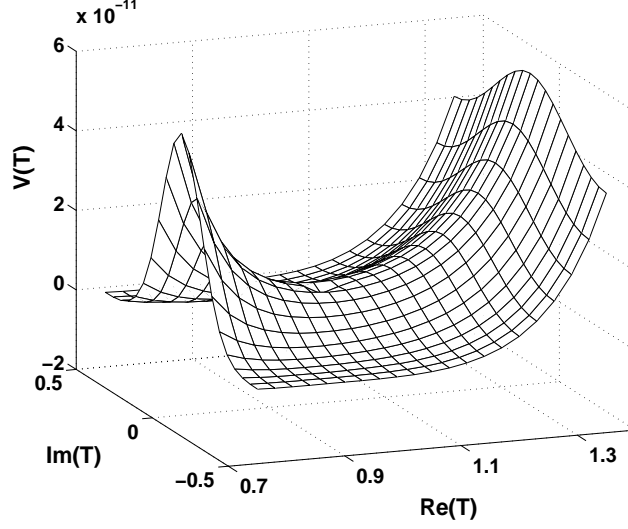


Figure 6: Scalar potential for $m = 1, n = 0, p(J) = 1, \beta = -0.3, V_{\text{dil}} = 0 = \delta_{\text{GS}}$.

approximation fails at a certain radius from $T = 1$. The effect on the scalar potential as a function of T is shown in fig. 7. At the values of T where a value for the scalar potential is not plotted, the gaugino condensate is destabilized by the $C_2''|z|^6$ term in the scalar potential. Here we appear to be on a branch of solutions with zero condensate and unbroken supersymmetry [39, 40] which is not connected to the rest of the surface $V(T)$. If the effective action for the gaugino condensate is valid down to $z = 0$ then there exists a supersymmetric, zero-energy vacuum for all T and it becomes a dynamical and cosmological question as to how the condensate becomes non-zero [40].

The minimum of the scalar potential is near $T = \rho$: this point is actually a (rather flat) *maximum* and the minima are close by inside the boundary of \mathcal{F} at $T = 0.8842 + 0.4844i$ and its images under modular transformations (and the complex conjugate of these values). The cosmological constant is negative. In this case $\hat{K}^T F^T / m_{3/2} = -0.0389 + 0.0136i$; we use this quantity as a rough measure of the amount of supersymmetry-breaking and CP violation that we expect to originate from a particular v.e.v. of T^1 . This

¹For $\delta_{\text{GS}} = 0$ we have $\hat{K}^T F^T / m_{3/2} = -3 - (T + T^*)(6d \ln \eta(T)/dT - d \ln H(T)/dT)$ (see eqs. 11–13); for non-zero δ_{GS} the analogous quantity is $\hat{K}^T F^T / m_{3/2} = -(1 - 3\delta_{\text{GS}}/b_0)(3 + 6(T + T^*)d \ln \eta(T)/dT) + (T + T^*)d \ln H(T)/dT$ (compare [10] and eq. 18).

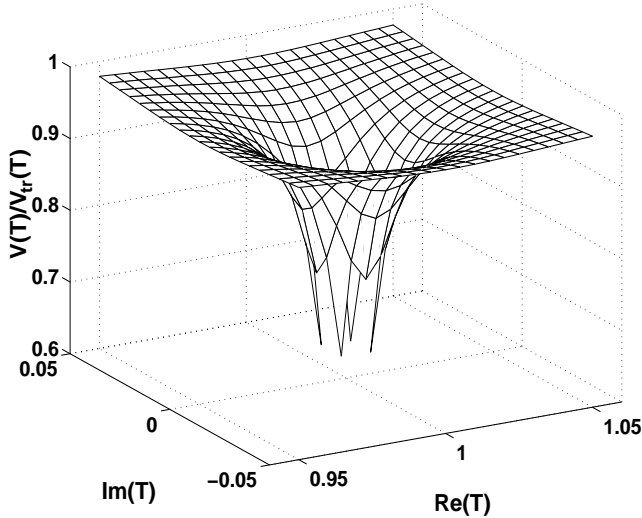


Figure 7: Scalar potential divided by the truncated potential near $T = 1$, for the same values of parameters as fig. 6.

result may be compared to the case where $m = 0$, $\delta_{\text{GS}} = -5$ and the potential is minimised at $T = 1.503$, which results in $\hat{K}^T F^T / m_{3/2} = -1.172$ — a factor of 50 larger. Where T is stabilised very near the self-dual point ρ (where F^T vanishes) with $m = 1$, F^T appears to be much smaller than at a general point inside \mathcal{F} , so its contribution to CP and flavour violation should be small. But we must ask, small relative to what? In order to have phenomenologically reasonable soft breaking terms we also require F^S to be non-zero, so that for F^T small we are near the dilaton-dominated limit of supersymmetry-breaking. Since $V_{\text{dil}} \propto |F^S|^2$ we should consider the effect of changing V_{dil} on the stabilization of T , as well as varying δ_{GS} and looking at the effects of deviations from the truncated approximation.

We first take $V_{\text{dil}} = 1.5$ while keeping $\delta_{\text{GS}} = 0$ and $\beta \equiv b_0 / (16\pi^2) = -0.3$ fixed: then the minimum is on the unit circle near $T = \rho$ at $T = 0.8777 + 0.4792i$ and $F^T = 0$. For $V_{\text{dil}} = 3$, the minimum is at $T = \rho$, $F^T = 0$ and the cosmological constant is tuned to zero.

Next consider changing δ_{GS} : we first take $\delta_{\text{GS}} = -15$, $V_{\text{dil}} = 0$, $\beta = -0.3$, in which case the minimum is on the boundary of \mathcal{F} at $T = 0.9058 + 0.5i$ and $F^T = -0.1075$ is real at the minimum. Considerations of modular invariance [22] indicate that this v.e.v. for T will not generate CP violation, however small deviations from universality may result. For $\delta_{\text{GS}} = -15$,

$V_{\text{dil}} = 3$, $\beta = -0.3$ the minimum is extremely close to $T = \rho$ and F^T vanishes. For $\delta_{\text{GS}} = 15$, $V_{\text{dil}} = 0$, $\beta = -0.3$ the minimum is on the unit circle at $T = 0.8873 + 0.4611i$ and F^T also vanishes; we find that this result is virtually unaffected by changing V_{dil} .

Finally for this case we consider the effect of corrections to the truncated approximation on the minimisation of the scalar potential. Since these scale with $|z_{\text{tr}}|^2 \propto e^{f_{\text{g}}/\beta}$, the corrections can be turned on or off by changing β (although only a finite range of values will result in a phenomenologically reasonable scale of supersymmetry-breaking). The truncated approximation corresponds to minimising the scalar potential in the limit $|z_{\text{tr}}| \rightarrow 0$, *i.e.* $\beta \rightarrow 0^-$, while the corrections are maximised for large, negative β .

We first take $\beta = -0.1$ to mimic the truncated approximation: for $V_{\text{dil}} = 0 = \delta_{\text{GS}}$ the minimum is at $T = 0.8842 + 0.4844i$ and $\hat{K}^T F^T / m_{3/2} = -0.0388 + 0.0134i$: differing only very slightly from the result at $\beta = -0.3$. For $\delta_{\text{GS}} = 0$, $V_{\text{dil}} = 3$ the minimum is again at $T = \rho$. Next we take the extreme large value $\beta = -0.9$, keeping $V_{\text{dil}} = 0 = \delta_{\text{GS}}$, which results in a minimum at $T = 0.8844 + 0.4847i$ with $\hat{K}^T F^T / m_{3/2} = -0.0403 + 0.0136i$. We see in this case that corrections to the truncated approximation do not much change the position of the minimum.

3.4.2 $m = n = 0$, $p(J) = J - 1$

This ansatz, equivalent to $m = 2$, $n = 0$, $p(J) = 1$ reproduces the desirable feature of the previous case of a small or zero contribution of the T modulus to CP violation and nonuniversality, without the “hole” where the gaugino condensate is destabilised. However, due to the zero of J at $T = 1$, the condensate vanishes smoothly at this point, an equally undesirable scenario for phenomenology! As for the case $m = 1$, $n = 0$, the phenomenologically interesting minima are near $T = \rho$. We start with the same set of parameters, $V_{\text{dil}} = 0 = \delta_{\text{GS}}$ and $\beta = -0.3$, for which the minimum is inside the fundamental domain at $T = 0.8754 + 0.4921i$ and $\hat{K}^T F^T / m_{3/2} = -0.0199 + 0.0071i$ and the cosmological constant is negative. For $V_{\text{dil}} = 3$ there is a minimum on the unit circle at $T = 0.8721 + 0.4894i$ with $F^T = 0$, and $T = \rho$ also appears to be a minimum.

In this case corrections to the truncated approximation may be significant. As we increase β , and thus the size of the corrections, the minimum inside the fundamental domain for $V_{\text{dil}} = 0$ moves towards the line $\text{Im}(T) = 1/2$, although only slowly. We may compare the values of $\hat{K}^T F^T / m_{3/2}$ at the

minimum: for $\beta = -0.3$ it is $0.87539 + 0.49207i$ and for $\beta = -0.9$ we have $\hat{K}^T F^T / m_{3/2} = 0.87547 + 0.49215i$. The effect is small, which follows from the fact that the corrections tend to be smallest around the minimum of the potential.

For $\delta_{\text{GS}} = -15$, $V_{\text{dil}} = 0$, $\beta = -0.3$ the modulus is stabilised at $T = 0.8857 + 0.5i$ and $\hat{K}^T F^T / m_{3/2} = -0.0538$. As β is increased to 0.7 the minimum remains on the line $\text{Im}(T) = 1/2$, so again the corrections do not seem to have a large effect. For $\delta_{\text{GS}} = +15$, $V_{\text{dil}} = 0$, $\beta = -0.3$ the minimum is on the unit circle at $T = 0.8774 + 0.4796i$ such that $F^T = 0$: this conclusion is also unchanged by increasing β and thus the corrections to truncation.

3.4.3 $m = 0$, $n = 1, 2, \dots$, $p(J) = 1$

In this case the scalar potential is minimised at $T = 1$ with $F^T = 0$, regardless of the values of other parameters. Near $T = \rho$ the same phenomenon occurs as in the case of $m = 1$, $n = 0$, $p(J) = 1$ near $T = 1$: at some finite radius away from the fixed point the truncated approximation fails and a non-zero value for the condensate becomes unstable. If we set $m = 0$, $n = 3$, $P(j) = 1$, which is equivalent to $m = n = 0$, $p(J) = J$, the supersymmetry-breaking minimum is again at $T = 1$ and the condensate goes smoothly to zero as $T \rightarrow \rho$.

3.4.4 $m = 1, 2$, $n = 1, 2$, $p(J) = 1$

In this case there is a region around both fixed points $T = 1$, $T = \rho$ where the condensate is destabilised or goes continuously to zero. For $m = n = 1$ and the parameter values $V_{\text{dil}} = 0$, $\delta_{\text{GS}} = 0$, $\beta = -0.3$, there are minima on the unit circle at $T = 0.9713 + 0.2378i$ (and at the complex conjugate value) with F^T vanishing (fig. 8). Again, the minimisation is robust to changes in parameters, in that the minimum remains on the unit circle with $F^T = 0$ when V_{dil} , δ_{GS} and β are changed within reasonable limits.

3.5 Interpretation of the results

The main results from minimization in any particular case are the value of T at the minimum and the size of the quantity $\hat{K}^T F^T / m_{3/2}$, which measures the contribution of the modulus to soft supersymmetry-breaking terms. Since the phase of F^T is not modular invariant, and the phases of the soft

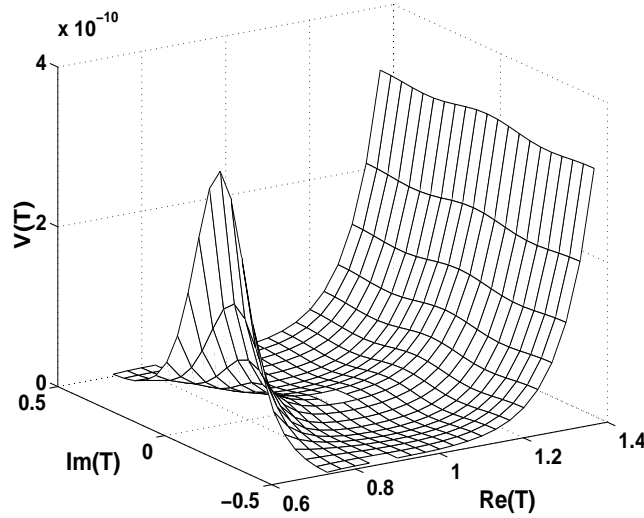


Figure 8: Scalar potential for $m = n = 1$, $p(J) = 1$, $\beta = -0.3$, $V_{\text{dil}} = 0 = \delta_{\text{GS}}$.

terms receive additional contributions from the T -dependence of the Yukawa couplings (which we have not explicitly calculated), we cannot give unambiguous measures of CP violation in the soft terms. However the value of $\langle T \rangle$ should allow us to diagnose whether CP is broken in the low-energy theory, and the size of F^T allows us to estimate how far we are from the dilaton- or moduli-dominated limits. The results presented seem to indicate that the most common patterns of supersymmetry-breaking are close to the dilaton-dominated limit (except when $V_{\text{dil}} = 0$ and $\eta(T)$ is the only modular form appearing in the threshold corrections): for various values of parameters, we obtain F^T vanishing or small compared to the scale of the gravitino mass, and when F^T is nonzero the v.e.v. of T is such that its contribution to supersymmetry-breaking may be either CP-conserving or (in the cases where $\langle T \rangle$ was in the interior of \mathcal{F}) CP-violating.

4 Further reflections

The existence of a viable phenomenology of CP violation in Yukawa couplings and soft terms depends largely on the dynamics of the T modulus in these models. The main contributions that the dilaton dynamics should make are to ensure the correct unified gauge coupling and, in the case where

$F^S \neq 0$, to contribute to soft terms in such a way as to alleviate the problems of nonuniversality and small gaugino masses in the moduli-dominated limit. We assumed, with [11, 12] that the dilaton plays essentially no role in observable CP violation or in the minimization in the T -direction. The results of [21] tend to validate this assumption: thus, while knowledge of the dilaton dynamics is needed to construct a full model, it is less important if one concentrates solely on CP. Thus the potentially interesting minima obtained here for the cases $m = 1, n = 0$ and $m = n = 1$ are reproduced in [21], up to a modular transformation (see their Table 2).

However, there are some cases where differences and unexpected results appear: namely, in the case $m = 2, n = 0$ where $F^S = 0$, and in the Kähler stabilization case. We find a SUSY-preserving minimum at $T = 1$, which is confirmed in [21] for all dilaton stabilization mechanisms, but we also find minima with negative vacuum energy near the self-dual point $T = \rho$, just as in the case $m = 1, n = 0$. This case (assuming a constant dilaton) was first considered in [45] in the context of cosmology. It is not clear what happens to these minima in the analysis of [21]. In the case with nonperturbative dilaton Kähler potential the property claimed in [38] of giving a nonzero F^S only survived in a single case, $m = n = 0$. One would like to know the reason for this apparent failure in more detail. This is somewhat of a crisis for the whole enterprise, since $F^S \neq 0$ is in general needed for acceptable phenomenology. Possibly a different choice of $P(y)$, or the combination of more than one hidden sector gauge group with a nonstandard Kähler potential, may resurrect this desirable feature.

The vanishing of F^S in this case presents an additional puzzle, since in the absence of this contribution to $V(S, T)$ the remaining part is universal (see eq. 18, where the first term is proportional to $|F^S|^2$, and eq. 19) or only depends on S via the values of $P(y)$ and its derivative at S_{\min} . Then one would expect an result for $\langle T \rangle$ similar to that for the racetrack and S -dual models, in which F^S always vanishes; however, no nontrivial values of T are actually found (Table 4). This may be due to the values of $P(y)$, $P'(y)$ being different from the corresponding perturbative formula: however, $P(y)$ only enters in the overall normalisation of the truncated potential, and P' through a small correction to the T -dependent part, so one would not expect large effects. The other possibility is that despite the exact constancy of S_{\min} when T is stabilized there is nontrivial variation over the T plane if S_{\min} is considered as a function of T . Without more data one cannot resolve this puzzle.

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